Benchmark on the 3D Numerical Modeling of a Superconducting Bulk

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AC losses are one of the key parameters in sizing High Temperature Superconducting (HTS) large scale devices. Therefore, their estimation has to be realistic in order to properly design the cryogenic system. Thus far, various 3D numerical models have been developed to that goal. However, the lack of analytical solutions in most of the cases and the scarcity of the experimental data leave the accuracy of the numerical methods an open question. In this paper, a benchmark on the 3D numerical modeling of a superconducting bulk is introduced. After a detailed description of the methods and their implementations, the results obtained by various independent teams are compared and discussed. This paper can be defined as a beginning of a conjoint work on 3D modeling of HTS bulks.

***Index Terms*— 3D modeling, AC losses, FEM, FVM, high temperature superconductors.**

I. INTRODUCTION

High Temperature Superconductors (HTS) are highly promising for applications requiring high power densities, e.g. when they are used as superconducting bulk magnets in motors [1] or bearings [2]. AC losses are one of the key factors to size properly the cryogenic system and therefore to ensure the reliability of HTS large scale applications. In the past years, various numerical models have been developed to calculate AC losses and simulate 3D problems involving HTS [3].

In practical applications, the assumptions required for analytical calculations are too restrictive and thus only numerical models can properly describe superconducting phenomenon [4]. It is therefore interesting to carry-out a comparative study between several numerical models in order to assess the validity of the results for a given problem. In this article, we introduce a new benchmark consisting of 3D simulations on an HTS cube. This problem is of interest due to its purely 3D nature and the presence of geometrical details such as edges and corners that can possibly lead to discrepancies between methods. This work gathers several international research groups involved in HTS numerical modeling.

In section II, we describe in details the benchmark problem. Section III gives an overview of the numerical models used by the different groups. Preliminary results are summarized in section IV.

II. BENCHMARK MODEL

A. Geometry

The studied geometry is an HTS cube of d = 10 mm side, as shown in Fig. 1. The air domain is delimited by a 100 mm side cube.

B. Material properties

The superconductor is modeled with an isotropic vector form linking the resistivity ρ to the current density vector **J** as:

$$\rho = \frac{E_c}{J_c}\left(\frac{|\mathbf{J}|}{J_c}\right)^{n-1} \qquad (1)$$

The parameters $J_c = 2.54.10^6$ A/m^2 and n = 23.3 have been chosen to fit experimental results obtained on Bi-2223 samples with E_c = 1 µV/cm.

C. Mesh

All the simulations have been made using the same mesh. The HTS cube was discretized with 12 elements on each edge of the cube. In the HTS cube, each tetrahedral element has a maximum side length of d/12. The mesh “cube_12” is composed of 71 797 elements with 32 838 elements in the HTS cube domain. *cube_12* was chosen as the best compromise between the AC losses calculation and the computation time based on parametric studies made on various mesh sizes.

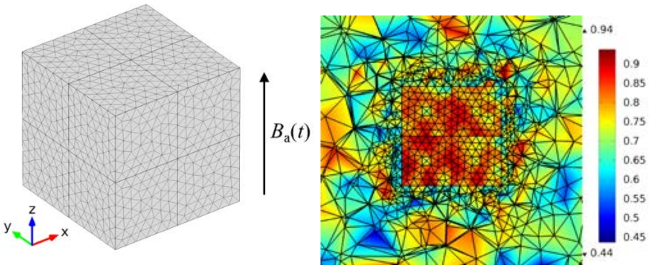

Fig. 1. Representation of the mesh *cube_12*: 3D overall mesh of the HTS cube on the left. Quality of the mesh in a *y-z* cut-plane at *x* = 0 on the right.

D. Study case

A uniform sinusoidal external magnetic flux density $B_a(t)$ at 50 Hz, with an amplitude B_{max} is applied to the cube along the z axis.

The variables of interest are the current density distributions inside the superconducting domain, and the instantaneous AC losses p_{AC} calculated by integrating the quantities **E** and **J** over the whole volume of the HTS cube as:

$$p_{\mathrm{AC}}(t) = \iiint_{\text{HTS cube}} \mathrm{E} \cdot \mathrm{J}\, dv \qquad (2)$$

Finally, we introduce P as the mean value of the instantaneous losses p_{AC} in the steady state regime.

III. NUMERICAL MODELS

The participants to the benchmark and the respective numerical methods are summarized in Table I. Formulations and codes (commercial or homemade) are given and more details will be added in the extended version of the paper. Moreover, novel techniques will be added and compared to the FEM with the *H*-formulation widely used by the HTS modeling community [5], [6].

TABLE I
NUMERICAL METHODS AND PARTICIPANTS INVOLVED IN THE BENCHMARK

Label	Method	Formulation	Code	Task leader
(B.1)	FEM	H	Daryl-Maxwell	G. Escamez
(B.2)	FEM	H	COMSOL(1)	K. Berger
(B.3)	FEM	H	COMSOL(2)	K. Berger
(B.4)	FEM	H	COMSOL(3)	L. Quéval
(B.5)	FEM	H	GetDP	A. Kameni
(B.6)	FVM	A-V	MATLAB	L. Alloui

In COMSOL 5.0, several methods can be used to implement an H formulation: (1) with the developed package called MFH physic, (2) by substituting *H* into *A* in the *A-V* formulation of the MF physic, (3) by manually implementing the constitutive equations with the PDE physic.

IV. FIRST TESTS

Considering the properties of the HTS cube, the full penetration flux density B_{p} can be calculated according to $B_{\mathrm{p}} = \mu_0 J_{\mathrm{c}} d/2 = 20$ mT [7]. A preliminary study was made with a $B_{\max} = 5$ mT. Fig. 2 displays the J_x component of the current density in the yz plane at $x = 0$. A macroscopic comparison is made between (B.1) and (B.4) and shows a good agreement.

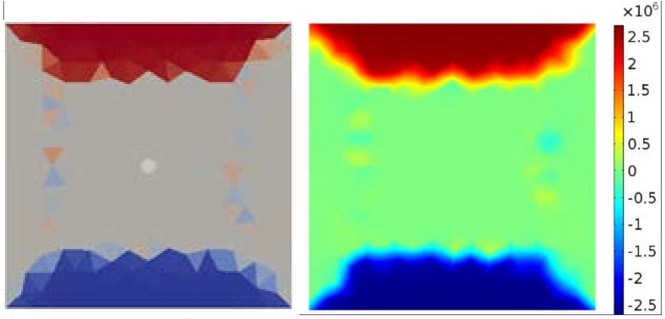

Fig. 2. J_x component of the current density in an yz plane at $x = 0$. On the left, with (B.1) and on the right with (B.4).

The instantaneous AC losses p_{AC} are shown in Fig. 3. Since these calculations are based on a time transient nonlinear simulation with zero initial conditions, it is necessary to simulate during a sufficiently long time to let the initial transient die out and reach a steady state which is achieved after 5 ms. The results are of the same order of magnitude. However, some differences can be noticed during the transient stage between 0 and 5 ms. These differences may be related to how the field is imposed in each model. This will be discussed in the final version of the paper.

The mean value of the AC losses in steady state and the computation time are given in Table II.

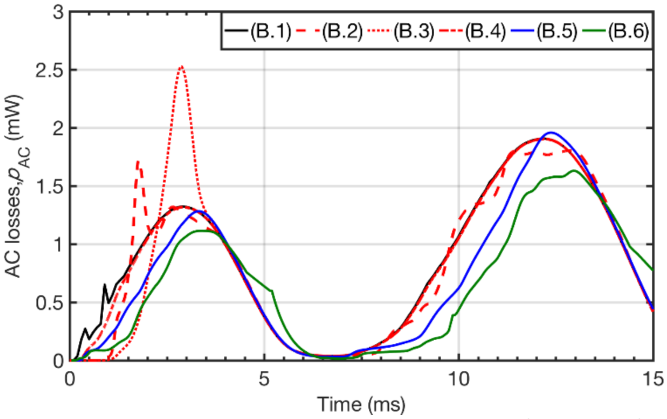

Fig. 3. Instantaneous AC losses p_{AC} versus time during the first 15 ms of the period for 5 different methods.

TABLE II
RESULTS FOR A MAXIMAL AMPLITUDE OF 5.10^{-3} T

Label	Time scheme	Comp. time	AC losses (mW)
(B.1)	Fixed (400 steps per period)	2h14	0.87
(B.2)	Adaptive	21h15	0.85
(B.3)	Adaptive	4h57	0.86
(B.4)	Adaptive	1h37	0.87
(B.5)	Fixed (400 steps per period)	1h26	0.75
(B.6)	Fixed (500 steps per period)	2h52	0.66

The computation times are given to have a rough idea and cannot be compared as all the simulations have been carried out on different platforms.

V. FUTURE WORK

After this preliminary study, the simulations will be extended with several meshes and applied fields. Macroscopic comparisons of the current density and magnetic field distributions will be investigated, in particular, the presence or the absence of a J_z component close to the edges of the cube.

Furthermore, simulations with pulsed magnetic fields applied will be carried out to investigate the behavior of the HTS bulk during the transient state.

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